ECONOMICS 113 - MATHEMATICAL ECONOMICS: GENERAL EQUILIBRIUM THEORY

What this course is about: Classic Arrow-Debreu general equilibrium model of the economy.

Economic General Equilibrium

General Equilibrium Theory: Who was Prof. Debreu and why did he have his own parking space in Berkeley's Central Campus??

Nobel Prizes: Arrow, Debreu

June 1993: A birthday party for mathematical general equilibrium theory!

October 2005: Mathematical Economics: The Legacy of Gerard Debreu http://emlab.berkeley.edu/users/cshannon/debreu/home.htm

What does mathematical general equilibrium theory do? Tries to put microeconomics on same basis of logical precision as algebra or geometry. Axiomatic method: allows generalization; clearly distinguishes assumptions from conclusions and clarifies the links between them.

Four ideas about writing an economic theory:

Ockam's razor (KISS - Keep it simple, stupid.), improves generality

Testable assumptions (logical positivism), avoids vacuity

Link with experience, robustness, Solow "All theory depends on assumptions which are not quite true. That is what makes it theory. The art of successful theorizing is to make the inevitable simplifying assumptions in such a way that the final results are not very sensitive. A "crucial" assumption is one on which the conclusions do depend sensitively, and it is important that crucial assumptions be reasonably realistic. When the results of a theory seem to flow specifically from a special crucial assumption, then if the assumption is dubious, the results are suspect. " (Contribution to the Theory of Economic Growth, 1956)

Precision, reliable results, Hugo Sonnenschein: "In 1954, referring to the first and second theorems of classical welfare economics, Gerard wrote 'The contents of both Theorems ... are old beliefs in economics. Arrow and Debreu have recently treated these questions with techniques permitting proofs.' This statement is precisely correct; once there were beliefs, now there was knowledge.

"But more was at stake. Great scholars change the way that we think about the world, and about what and who we are. The Arrow-Debreu model, as communicated in *Theory of Value* changed basic thinking, and it quickly became the standard model of price theory. It is the 'benchmark' model in Finance, International Trade, Public Finance, Transportation, and even macroeconomics. ... In rather short order it was no longer 'as it is' in Marshall, Hicks, and Samuelson; rather it became 'as it is' in *Theory of Value*." (remarks at the Debreu conference, Berkeley, 2005).

Requirements: There will be weekly problem sets, two midterms, and a take-home portion of the final exam. Feel free to co-operate with friends and classmates on problem sets.

All examinations are open-book, open-notes. Confidentiality is required during examinations. Please strictly observe academic integrity. Examinations should be your own personal work. During examinations, other people (classmates, friends, professors --- except the TA and Prof. Starr) are CLOSED; do not discuss examination materials until after the exam has been collected.

Examination Schedule:

Midterm 1 (covers syllabus sections 1 to 5). In Class, date TBA.

Midterm 2 (covers syllabus sections 1 to 11). In Class, date TBA and Take Home due TBA.

Final: There will be a take home section of the final exam, due date TBA. In-class final exam is scheduled for Wednesday June 11 11:30 a.m. – 2:30 p.m.

Grading: Problem sets, 5%; midterm 1, 15%; midterm 2, 30%; final exam, 50%. Additional credit for class participation.

Prerequisites: A year of calculus and a year of upper division microeconomic theory (at UCSD these courses are Math 20 A-B-C, and Economics 100A-B or 170A-B). The prerequisites may be taken concurrently. Students with very strong mathematics preparation (typically including one quarter of real analysis, UCSD Math 140A or 142A) may enroll without economics prerequisites.

Text: R. Starr's *General Equilibrium Theory: An Introduction* , Cambridge University Press, 1997. Available in paperback from campus bookstore and from amazon.com. Update Starr with corrigenda.

Reserve Materials: The following items have been requested on reserve in the Geisel library:

Arrow, K. J. and F. H. Hahn, *General Competitive Analysis*

Arrow, Kenneth J., "A Difficulty in the concept of social welfare", Journal of Political Economy, 58 (1950), pp. 328 - 346. Reprinted in Arrow and Scitovsky, eds., Readings in Welfare Economics, 1969.

Bartle, R., *The Elements of Real Analysis* , 1st edition, 1964

Bartle, R. and D. R. Sherbert, *Introduction to Real Analysis* , 2nd edition, 1992 and 3rd edition, 2000

Cornwall, R. R., *Introduction to the Use of General Equilibrium Analysis*

Debreu, G., *Theory of Value*

Eatwell, J., M. Milgate, and P. Newman (eds.) *The New Palgrave: General Equilibrium*

Quirk, J. and R. Saposnik, *Introduction to General Equilibrium and Welfare Economics*

Starr, R. M., *General Equilibrium Theory: An Introduction*

Varian, H., *Microeconomic Analysis, 3rd ed.* , 1992

McCloskey, D. "The Futility of Blackboard Economics" in *The Vices of Economists--The Virtues of the Bourgeoisie,* Amsterdam University Press, 1996.

Gibbard, A. and H. Varian, "Economic Models" *Journal of Philosophy*, v. 75, 1978, pp. 664-677.

TOPIC OUTLINE

Lectures will closely follow Starr's *General Equilibrium Theory: An Introduction* . Please read the relevant portion of Starr's *General Equilibrium Theory* before the topic is covered in class.

Scheduled holiday: Monday, May 25.

Introduction

1. The simplest general equilibrium model: Robinson Crusoe (3 lectures)
   Starr, 1.1, 1.2

2. The Edgeworth Box (2 lectures)
   Starr, 1.3

3. A simple demonstration of existence of general equilibrium (1 lecture)
   Starr, 1.4
   "Kenneth J. Arrow (1921 -)" by R. Starr
   Optional: Arrow-Hahn, chaps.1, 2

Cornwall, 1.1, 1.2, 1.3
Geanakoplos, John, "Arrow-Debreu Model of General Equilibrium" in *The New Palgrave: General Equilibrium*
Varian, 17.1 - 17.5

Mathematics

4. Set notation, Euclidean N-dimensional space, R^N (1 lecture)
Starr, 2.1 "Set Theory"
Starr, 2.4 "R^N, Real N-dimensional Euclidean Space"
Optional: Bartle, Section 1, 7, 8, 11
Bartle and Sherbert, 2nd edition section 1.1, chap. 2, sections 3.1, 3.2, 3.3, chap.10; 3rd ed. section 1.1, chap. 2, sections 3.1, 3.4, 11.1, 11.2
Debreu, 1.2, 1.6, 1.9a - 1.9f

5. Continuous Functions (1 lecture)
Starr, 2.3 "Functions,"
2.5 "Continuous Functions"
Optional: Bartle, Sections 2, 15
Bartle and Sherbert, 2nd ed., sections 5.1, 5.2, 5.3; 3rd ed. sections 5.1, 5.2, 5.3, 11.3 Debreu, 1.3, 1.8

Midterm 1: will cover topics 1 – 5

6. Convexity (1 lecture)
Starr, 2.6 "Convexity"
Optional: Debreu, Section 1.9

The Arrow-Debreu Model of Economic General Equilibrium

7. Representation of Commodities and Prices, Firms and Producers (2 lectures)
Starr, chap. 3, 4.
Optional: Debreu, Chapter 2, 3
Geanakoplos "Arrow-Debreu Model of General Equilibrium" in *New Palgrave*.
Quirk and Saposnik, 1.7, 2.1, 2.3
Arrow-Hahn, Chapter 3

8. Households, Consumers (3 lectures)
Starr, chaps. 5, 6

Optional: Debreu, Chapter 4
Cornwall, Section 1.4
Quirk and Saposnik, 1.5, 1.6
Arrow-Hahn, 4.1-4.3
Varian, 7.1, 7.2

9. Brouwer Fixed Point Theorem (1 lecture)
Starr, 2.7 "Brouwer Fixed Point Theorem"
Optional: Debreu, Section 1.10
Nikaido, "Fixed Point Theorems" in *New Palgrave: General Equilibrium*.

10. Equilibrium (2 lectures)
Starr, chap. 7
Optional: Debreu, Chapter 5
Cornwall, Section 1.6
Quirk and Saposnik, 1.7, 2.1, 2.3
Arrow-Hahn, Chapter 5
Debreu, "Existence of General Equilibrium," *New Palgrave: General Equilibrium*
McKenzie, "General Equilibrium," *New Palgrave: General Equilibrium*

Midterm Exam 2 based on topics 1 -10

Welfare Economics

11. Separation Theorems (1 lecture)
Starr, 2.8 "Separation Theorems"
Optional: Debreu, Section 1.9.v - 1.9.x
Cornwall, Section 8.1.4
Varian, 26.11

12. Fundamental Theorems of Welfare Economics (3 lectures)
Starr, chap. 12
Optional: Debreu, Chapter 6
Cornwall, Sections 4.1, 4.2, 4.3, 4.5
Quirk and Saposnik, 4.4, 4.5
Varian, 17.6, 17.7.

13. The Arrow Possibility Theorem (3 lectures)
    Readings TBA

Extending the General Equilibrium Model

15. Equilibrium over Time: Futures Markets (1 lecture)
    Starr, 15.1 "Introduction", 15.2 "Time: Futures Markets"

16. Constant Returns and U-Shaped Cost Functions (1 lecture)
    Starr, 16.7 "Kakutani Fixed-Point Theorem"
    Additional notes TBA

Partial and General Economic Equilibrium

PARTIAL EQUILIBRIUM

$S_k(p^o{}_k) = D_k(p^o{}_k)$, with $p^o{}_k > 0$ (or possibly, $p^o{}_k = 0$), or

$p^o{}_k = 0$ if $S_k(p^o{}_k) > D_k(p^o{}_k)$.

GENERAL EQUILIBRIUM For all i = 1,...,N,

$D_i(p^o{}_1, p^o{}_2,...,p^o{}_N) = S_i(p^o{}_1,..,p^o{}_N)$, $p^o{}_i > 0$, and

$p^o{}_i = 0$ for goods i such that

$$D_i(p^o{}_1,...,p^o{}_N) < S_i(p^o{}_1,...,p^o{}_N).$$

What's wrong with partial equilibrium? Suppose there's no consistent choice of $(p^o{}_1,..,p^o{}_N)$. Then there would be (apparent) partial equilibrium --- viewing each market separately --- but no way to sustain it, because of cross-market interaction. Competitive equilibrium is supposed to make efficient use of resources by minimizing costs and allowing optimizing consumer choice. But how do we know prices in other markets reflect underlying scarcity assuming "other things being equal" . If not, then apparently efficient equilibrium allocation may be wasteful. A valid notion of equilibrium and efficiency needs to take cross-market interaction into account.

Three big ideas

Equilibrium: S(p) = D(p)
Decentralization
Efficiency

The Robinson Crusoe Model

q = oyster production

c = oyster consumption
168 (hours per week) endowment
L = labor demanded
R = leisure demanded
168-R =labor supplied

$$q = F(L) \tag{1.1}$$

$$R = 168 - L \tag{1.2}$$

Centralized Allocation

We assume second order conditions so that local maxima are global maxima:

$$F'' < 0, \frac{\partial^2 u}{\partial c^2} < 0, \frac{\partial^2 u}{\partial R^2} < 0.$$

$$u(c,R) = u(F(L), 168 - L) \tag{1.3}$$

$$\max_L \; u(F(L),\ 168 - L) \tag{1.4}$$

$$\frac{d}{dL}\, u(F(L),\ 168 - L) = 0 \tag{1.5}$$

$$u_c F' - u_R = 0 \tag{1.6}$$

$$\left[-\frac{dq}{dR}\right]_{u = u\max} = \frac{u_R}{u_c} = F' \tag{1.7}$$

Pareto efficient

$$MRS_{R,c} = MRT_{R,q} (= RPT_{R,q}\ )$$

Decentralized Allocation

$$\Pi = F(L) - wL = q - wL \tag{1.8}$$

Income:

$$Y = w \cdot 168 + \Pi \tag{1.9}$$

Budget constraint:

$$Y = wR + c \tag{1.10}$$

Equivalently, c = Y- wR = Π + wL = Π + w(168-R), a more conventional definition of a household budget constraint.

Firm profit maximization:

$$\Pi = q - wL \tag{1.11}$$

$$\frac{d\Pi}{dL} = F' - w = 0, \text{so } F'(L^0) = w \tag{1.13}$$

Household budget constraint:

$$wR + c = Y = \Pi^0 + w168 \tag{1.14}$$

Choose c, R to maximize u(c, R) subject to (1.14). The Lagrangian is

V = u(c,R) - λ (Y - wR - c)

$$\frac{\partial V}{\partial c} = \frac{\partial u}{\partial c} + \lambda = 0$$

$$\frac{\partial V}{\partial R} = \frac{\partial u}{\partial R} + \lambda w = 0$$

Dividing through, we have

$$\text{MRS}_{R,c} = \left[-\frac{dc}{dR}\right]_{u=constant} = \frac{\frac{\partial u}{\partial R}}{\frac{\partial u}{\partial c}} = w \tag{1.18}$$

$$wR + c = w168 + \Pi^0 \tag{1.19}$$

$$c = w(168 - R) + \Pi^0 \tag{1.20}$$

<u>Walras' Law</u>

Note that the Walras Law holds at all wage rates --- both in and out of equilibrium. It is not an equilibrium condition.

$$Y = w \cdot 168 + \Pi = w168 + q - wL \quad = wR + c \qquad (1.21)$$

$$0 = w(R - (168 - L)) + (c - q) \qquad (1.22)$$

$$0 = w(R + L - 168) + (c - q)$$

Definition : Market equilibrium. Market equilibrium consists of a wage rate w^0 so that at $w^0, q = c$ and $L = 168 - R$, where q, L are determined by firm profit maximizing decisions and c, R are determined by household utility maximization. (in a centralized solution L=168-R by definition; in a market allocation wages and prices should adjust so that as an equilibrium condition L will be equated to 168-R).

Profit maximization at w^0 implies $w^0 = F'(L^0)$. (Recall (1.13))
Utility maximization at w^o implies

$$\frac{u_R(c^0, R^0)}{u_c(c^0, R^0)} = w^0 \qquad (1.23) \quad \text{(Recall (1.18))}$$

Market-clearing implies $R^0 = 168 - L^0, c^0 = F(L^0)$.

So combining (1.13) and (1.23), we have

$$F' = \frac{u_R}{u_c} \qquad (1.24)$$

which implies Pareto efficiency.